

MIXED CONVECTIVE MICROPOLAR FLUID FLOW WITH MASS TRANSFER AND HEAT SOURCE ACROSS AN EXPONENTIALLY EXTENDING SHEET IN MHD

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ABSTRACT

The topic of MHD mixed convective micropolar fluid flow with mass transfer over an exponentially extending porous sheet is examined in the current paper. After undergoing a similarity transformation, the flow controlling equations were converted to an ODE and solved using the finite differences approach. All non-dimensional characteristics, including the material constant K , magnetic parameter M , Prandtl number Pr , heat source parameter S , Schmidt number Sc , and chemical reaction parameter γ , are displayed graphically in order to examine the regulating parameters of the velocity, temperature, and concentration profiles. Understanding thermal and chemical reaction behaviors toward non-Newtonian fluids is provided by this work.

KEYWORDS: MHD, Mixed convection, ODE, finite difference method, non-Newtonian fluid

1. INTRODUCTION

Recently, most of researchers have fascinated tremendous attention towards the theory of micropolar fluid and interest of extensive research has given towards the study of MHD micropolar fluid over a stretching sheet due to its industrial applications. Chaudary et al.¹ investigated the heat and mass transfer processes on an unsteady flow of a micropolar fluid past through a porous medium bounded by a semi-infinite vertical plate. Faiz Awad et al.² studied Dufour and Soret effects of a micropolar fluid in a horizontal channel. In the above article a couple of partial differential equations were solved analytically by Homotopy Analysis Method and numerically by bvp4c MATLAB. Mohamed Abd-El-aziz et al.³ discussed the effect of variable viscosity and variable thermal conductivity of an unsteady forced convective flow and heat transfer characteristics of a viscoelastic liquids film past on a horizontal stretching sheet in presence of viscous dissipation. Dakshinamoorthy et al.⁴ reported the steady, two-dimensional, boundary layer flow of an electrically conducting viscous incompressible fluid past in a continuously moving surface in presence of uniform transverse magnetic field. Kishore et al.⁵ presented MHD viscous incompressible fluid past in an oscillating vertical plate fixed in a porous medium in presence of

variable heat and mass diffusion, radiation and viscous dissipation.

Further, Roja et al.⁶ investigated the effect of first-order homogeneous chemical reaction and thermal radiation on hydromagnetic free convective micropolar fluid flow through semi-infinite vertical moving porous plate. Rajasekaret al.⁷ studied the influence of variable viscosity and thermal conductivity on an unsteady MHD, two-dimensional laminar viscous flow of an incompressible electrically conducting fluid past through a semi-infinite vertical plate with mass transfer processes. Shehzad et al.⁸ discussed MHD flow of Casson fluid over a porous stretching sheet in presence of a chemical reaction. Shit et al.⁹ reported the influence of thermal radiation and temperature dependent viscosity on a free convective flow and mass transfer characteristics of an electrically conducting fluid over an isothermal stretching sheet. Yigit Aksoy et al.¹⁰ analyzed the new techniques in perturbation iteration method for heat transfer problems. This new technique is practically applied in much nonlinear heat transfer and flow problems and is successfully implemented. Sankar Reddy et al.¹¹ discussed MHD flow of a micropolar fluid past in semi-infinite vertically moving porous plate fixed in a porous medium in presence of thermal radiation, thermal diffusion, and the first order homogeneous chemical reaction. Mohammed Abd

El-Aziz¹² investigated the unsteady mixed convective flow of a viscous incompressible micropolar fluid close to a heated vertical surface with viscous dissipation where the buoyancy force assists or opposes the flow.

Moreover, Najwa Najibet.al¹³ reported the stagnation point flow and mass transfer characteristics through a stretching/shrinking cylinder. Hayatet.al¹⁴ discussed on steady magneto-hydrodynamic (MHD) flow of viscous nanofluid over an permeable exponentially stretching surface in a porous medium with convective boundary condition. Lavanyaet.al¹⁵ analyzed two-dimensional steady MHD free convective flow past in a vertical porous plate in a porous medium in presence of thermal radiation, chemical reaction. Bhim Sen Kalaet.al¹⁶ discussed the influence of first-order chemical reaction and oscillatory suction on magnetohydrodynamic (MHD) flow of a viscous incompressible electrically conducting fluid through a porous medium in presence of transverse magnetic field. Prabir Kumar Kunduet.al¹⁷ studied hydrodynamic free convective micropolar fluid flow in a rotating frame of reference with constant wall heat and mass transfer in a porous medium which was bounded by a semi-infinite porous plate. Abdul Rehman.et.al¹⁸ analyzed the influence of natural convective heat transfer analysis for a steady boundary layer flow of an Eyring Powell fluid which was flowed through a vertical circular cylinder. Sethet.al¹⁹ discussed unsteady hydromagnetic natural convective heat and mass transfer processes of a viscous incompressible, electrically conducting chemically reactive and optically thin radiating fluid past in an exponentially accelerated moving vertical plate with arbitrary ramped temperature fixed in a fluid-saturated porous medium.

Numerical analysis of a steady two-dimensional hydromagnetic stagnation point flow of an electrically conducting nanofluid past in a stretching surface with induced magnetic field was studied by Gireshaet.al²⁰. KhairyZaimiet.al²¹ discussed the influence of partial slip on stagnation point flow and heat transfer processes towards a stretching vertical sheet and the problem was solved numerically by using the shooting method. Sheri Siva Reddyet.al²² analyzed boundary layer analysis on an unsteady MHD free convective micropolar fluid flow past in a semi-infinite vertical porous plate with the presence of diffusion Thermo, heat absorption, and homogeneous chemical reaction. Shahirah Abu Bakeret.al²³ investigated a mathematical analysis of forced convective boundary layer stagnation-point slip flow in Darcy-Forchheimer porous medium through a shrinking sheet. Sahin.Ahmad.et.al²⁴ analyzed perturbation

analysis of combined heat and mass transfer processes in MHD steady mixed convective flow of an incompressible, viscous, Newtonian, electrically conducting and chemical reacting fluid past over an infinite vertical porous plate in presence of the homogeneous chemical reaction of first order.

Shamshuddinet.al²⁵ discussed the heat and mass transfer processes on the unsteady incompressible MHD flow of chemically reacting micropolar fluid flow to a vertical porous plate fixed in a saturated homogeneous porous medium in presence of radiation effect and Joule heating. Okechiet.al²⁶ investigated the boundary layer analysis of viscous fluid flow induced by rapidly stretching curved surface with an exponential velocity. Alet.al²⁷ investigated MHD boundary layer nanofluid flow along a moving wedge. Arunaet.al²⁸ studied parameterized perturbation method (PPM) for the solution of nonlinear equations which was arising in heat transfer processes. Abid Hussananet.al²⁹ investigated a closed form solution for an unsteady free convective flow of a micropolar fluid past over a vertical plate oscillating in its own plane with a Newtonian heating condition. Sivakumar Narsuet.al³⁰ analyzed the effect of thermo diffusion in unsteady MHD combined convective boundary layer flow of a Kuvshinski fluid through vertical porous plate in slip flow regime.

A finite element computational analysis was done for MHD double-diffusive mixed convective micropolar fluid flow to a vertical porous plate fixed in the saturated porous medium by Shamshuddin.et.al³¹. Dulal palet.al³² studied the double-diffusive heat and mass transfer processes of an oscillatory viscous electrically conducting micropolar fluid past over a moving plate with convective boundary condition and chemical reaction. Chandra sekaret.al³³ investigated MHD free convective flow past in an inclined porous stretching sheet with a presence of viscous dissipation and radiation effect. Sivakamiet.al³⁴ studied perturbation technique for an unsteady MHD free convective flow of two immiscible fluid in a horizontal channel with the influence of Dufour effect in presence of chemical reaction and heat source. Zahir Shahet.al³⁵ discussed micropolar nanofluid flow of Casson fluid past between the two rotating parallel plates with the influences of hall currents and thermal radiation. Motivation by the above studies, the novelty of present article extends the work of Anuradha³⁶ to MHD mixed convective micropolar fluid flow with mass transfer over an exponentially stretching porous sheet.

2. MATHEMATICAL FORMULATION

The steady two dimensional, incompressible, MHD mixed convective micropolar fluid flow through an exponentially stretching sheet is considered with heat source and chemical reaction. Assume that the sheet is being stretched with exponential velocity $U_w = ae^{x/L}$; $a > 0$; a is a stretching constant. The physical configuration and Cartesian coordinate system are taken as follows: Leading edge of the sheet is located in the origin, considerable x axis in positive direction and along the stretching sheet, positive y axis is normal to the stretching sheet, Cartesian coordinate axes (x, y, z) are with corresponding velocities (u, v, θ) , the uniform magnetic field B_0 is applied towards the positive direction of y axis and Dufour and Soret effects are negligible in the concentration of mass. Under the Boussinesq and boundary layer approximation, the governing equations can be written as follows

thermal expansion coefficient, ν is the kinematic viscosity, μ is the dynamic viscosity, ρ is the density, N is the microrotation, j is the micro inertia per unit mass, γ is the spin gradient viscosity, k is the vortex viscosity, T is the temperature in the boundary layer, C is the concentration in the boundary layer, L is the reference length, D_m is the modular diffusivity,

Assume that the exponential stretching sheet expression for U_∞ , U_w , T_w and C_w from the stagnation point flow are defined as

$$U_\infty = ae^L, -$$

$U_w = be^L$, The equations (1)-(5) with the boundary conditions (6) are transformed into ordinary differential equations by using similarity transformation. We introduce the following similarity variables and non-dimensional variables (8) to the equations (1)-(6), Equation (1) is satisfied by Cauchy Riemann equations and the equations (2)-(5) becomes

3. NUMERICAL PROCEDURE, RESULTS AND DISCUSSION

To solve this mathematical model, firstly, the given system of partial differential equations can be transformed into set of coupled non-linear boundary layer equations (9)-(12) with associated boundary condition (13) by using similarity transformation and then the equations (9)-(12) solved

numerically by finite difference method. To study the flow characteristics and mixed convective parameters, the physical situation of non-dimensional parameters on velocity, temperature and concentration profiles are analyzed numerically and presented with the help of graphs. This study is extension of Anuradha(2018) for MHD mixed convective boundary layer flow of micropolar fluid along exponentially Stretchingsheet with chemical reaction and heat source. The non-dimensional parameters such as Material constant K , magnetic parameter M , Prandtl number Pr , Heat source parameter S , Schmidt number Sc , and chemical reaction parameter γ on velocity, temperature and concentration profiles are presented with help of graphs. For the depth of analysis, Assisting flow $\lambda > 1$, $\delta > 1$ and Opposing flow $\lambda < 1$, $\delta < 1$ are taken into account.

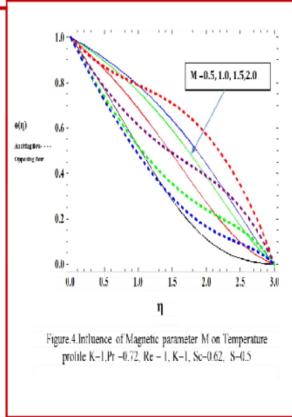
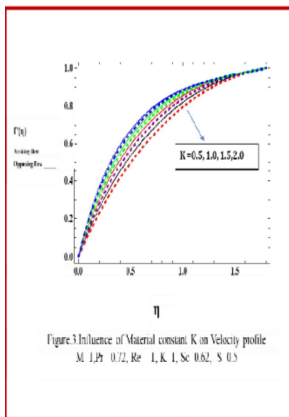
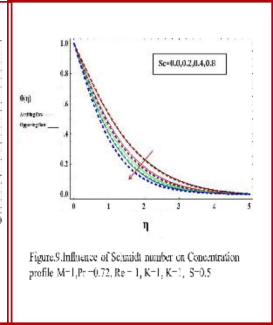
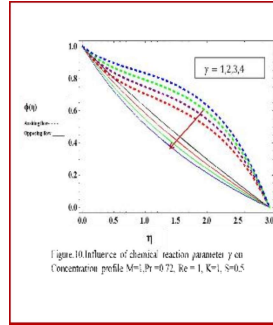
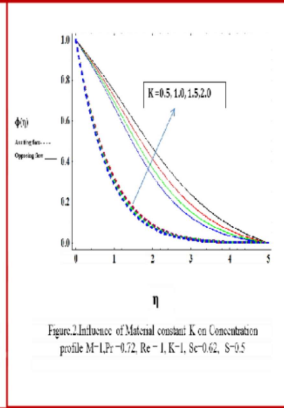
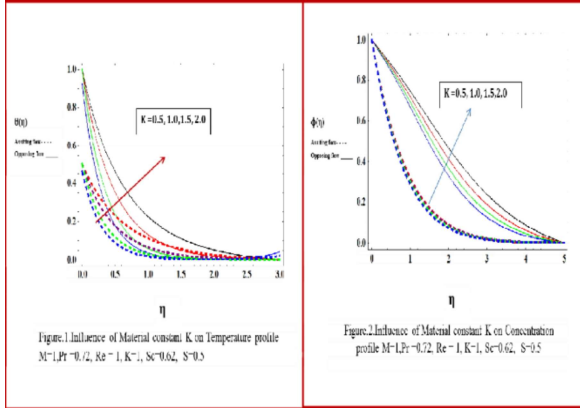
Figures 1-3 demonstrate the variation of Material constant K on temperature, concentration and velocity profiles respectively. Increasing values of Material constant K decrease the velocity profile for both assisting and opposing flows. Whereas increasing values of Material constant K decrease the concentration and temperature profiles. This proves the impact that the thickness of thermal boundary layer has increased for assisting flow comparing with effect of opposing flow.

Figure 4-6 depict the effect of magnetic parameter M on temperature, concentration and velocity profiles respectively. Increasing values of magnetic parameter M reduce temperature and concentration profiles and increasing values of magnetic parameter M enhances the velocity field.

Figure 7 and 8 exhibit the effect of Prandtl number Pr and heat source parameter S on temperature profile respectively. Increment in the values of Prandtl number Pr decreases the

temperature profile for both assisting and opposing flows. Increasing values of heat source parameter S increase temperature profile which shows that increasing values of heat source parameter S increase the level of temperature for both flows in micropolar fluid.

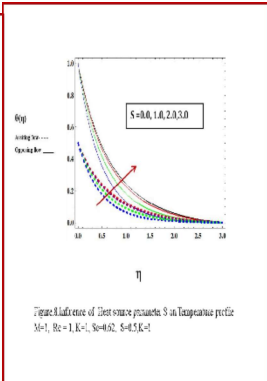
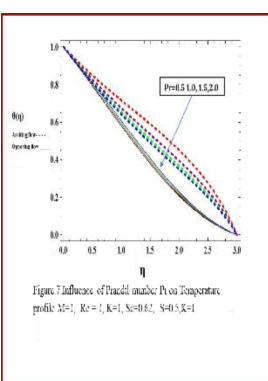
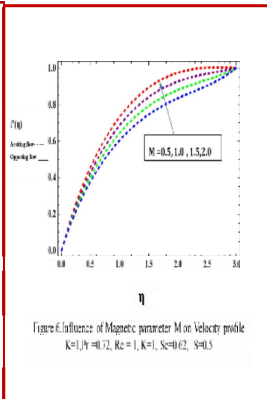
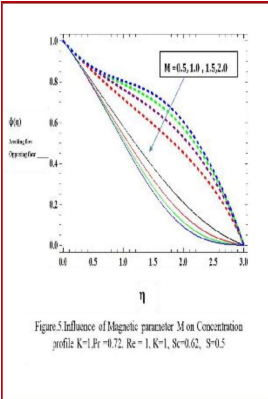
Figures 9 and 10 illustrate the effect of Schmidt number Sc and chemical reaction parameter γ on concentration profile respectively. Increasing values of Schmidt number Sc and chemical reaction parameter γ reduces concentration profile. Increasing values of Schmidt number Sc improves the concentration level in micropolar fluid flow.



4. CONCLUSION

This present article studied MHD mixed convective micropolar fluid flow with mass transfer and heat source over an exponentially stretching porous sheet. The conclusions are as follows:

- Increasing values of Material constant K decrease the velocity profile for both assisting and opposing flows. Whereas increasing values of Material constant K decrease the concentration and temperature profiles. This proves the impact that the thickness of thermal boundary layer has increased for assisting flow comparing with effect of opposing flow.
- Increasing values of magnetic parameter M reduce temperature and concentration profiles and increasing values of magnetic parameter M enhances the velocity field.
- Increment in the values of Prandtl number Pr decreases the temperature profile for both assisting and opposing flows. Increasing values of heat source parameter S increase temperature profile which shows that increasing values of heat source parameter S increase the level of temperature for both flows in micropolar fluid.
- Increasing values of Schmidt number Sc and chemical reaction parameter γ reduces concentration profile. Increasing values of Schmidt number Sc improves the concentration level in micropolar fluid flow.



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