

# Numerical Study of Free Convection Fluid Flow over a Vertical Porous Plate with Induced Magnetic Field in Magneto-hydro dynamics (MHD)

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**Abstract:** Examine a two-dimensional unsteady Magneto hydro dynamics (MHD) free convection flow of viscous, incompressible, and electrically conducting fluid past a vertical plate in this paper while taking into account the Grashof, Modified Grashof, Prandtl, Schamidl, and Dufour numbers. The problem's governing equations, which comprise a system of coupled non-linear ordinary differential equations and a system of non-linear partial differential equations, are numerically solved using the widely used explicit finite difference approach. A widely utilized method for examining general nonlinear partial differential equations is the finite-difference method. Numerous areas of applied mathematics, including hydrodynamics, elasticity, and quantum physics, involve partial differential equations. Therefore, the purpose of the suggested study is to investigate the numerical results which are performed for various physical parameters such as velocity profiles, temperature distribution and concentration profiles within the boundary layer are separately discussed in graphically.

**Keyword:** Explicit Finite Difference Method, Mass and Heat Transfer, Non-Linear PDE, Rotating System, and MHD vertical isothermal non- conducting plate with variable suction and internal heat generation in the presence of

## Introduction

MHD boundary layer flow has become significant applications in industrial manufacturing processes such as plasma studies, petroleum industries Magneto hydrodynamics power generator cooling of clear reactors, boundary layer control in aerodynamics. Many authors have studied the effects of magnetic field on mixed, natural and force convection heat and mass transfer problems.

S. Idowu et al [1] studied the radiation effect on unsteady heat and mass transfer of MHD and dissipative fluid flow past a moving vertical porous plate with variable suction in the presence of heat generation and chemical reaction. M. S. Alam et al [2] studied the free convective heat and mass transfer flow past an inclined semi infinite heated surface of an electrically conducting and steady viscous incompressible fluid in the presence of a magnetic field and heat generation. Mohammad Shah Alam et al [3] mass transfer over an inclined stretching sheet in the presence of a uniform magnetic field. M. Umamaheswar et al and [4] reported an unsteady magneto hydrodynamic free convective, Visco-elastic, dissipative fluid flow embedded in porous medium bounded by an infinite inclined porous plate in the presence of heat source, P. R. Sharma et al [5] investigated the flow of a viscous incompressible electrically conducting fluid along a porous

transverse magnetic field. Hemant Poonia and R. C. Chaudhary [6] analyzed the heat and mass transfer effects on an unsteady two dimensional laminar mixed convective boundary layer flow of viscous, incompressible, electrically conducting fluid, along a vertical plate with suction, embedded in porous medium, in the presence of transverse magnetic field and the effects of the viscous dissipation. C. V. Ramana Kumari and N. Bhaskara Reddy [7] reported an analytical analysis of mass transfer effects of unsteady free convective flow past an infine vertical porous plate with variable suction Reddy et al [8]. K. Bhagya Lakshmi et al [9] investigated the hydromagnetic effects on the unsteady free convection flow, heat and mass transfer characteristics in a viscous, incompressible and electrically conducting fluid past an exponentially accelerated vertical plate and the heat due to viscous dissipation roll of magnetic field on ionized Magnetohydrodynamic fluid flow through an infinite rotating vertical porous plate with heat transfer. parallel porous plates under the influence of inclined magnetic field with heat transfer. Consider the thermal radiation interaction with unsteady MHD flow past rapidly moving plate has a great important application in different brance of science and to the chemical engineering processes and in many technological fields. This types of problems were studied by

Abdulwaheed Jimoh[16], Rasualizadehand Alirezadarvish [17]. Numerical solution of MHD fluid flow past an infinite vertical porous plate was done by K. Anitha [18]. Takhar and Ram [19] studied the effects of Hall current on hydro-magnetic free convective flow through a porous medium. Chaudhary and Sharma [20] have analytically analyzed the steady combined heat and mass transfer flow with induced magnetic field. The aim of this paper is to investigate numerically transient MHD combined heat and mass transfer by mixed convection flow over a continuously moving vertical porous plate under the action of strong magnetic field taking into account the induced magnetic field with constant heat and mass fluxes. The governing equations of the problem contain a system of partial differential equations which are transformed by usual transformation into a non-dimensional system of partial coupled non-linear differential equations. The obtained non-similar partial differential equations will be

**1. Mathematical Model of the Flow**

MHD power generation system combined heat and mass transfer in natural convective flows on moving vertical porous plate with thermal diffusion is considered. Let, the x-axis is chosen along the porous plate in the direction of flow and the y-axis is normal to the plate. The MHD transfer flow under the action of a strong magnetic field. The form of the induced magnetic field is  $(B_x, B_y, 0)$ . Now the Maxwell's equation is  $\nabla \cdot B = 0$  so the magnetic field becomes  $B_y = B_0$ .

Initially, consider that the plate as well as the fluid are at the same temperature  $(= T_\infty)$  and the concentration level

$(= C_\infty)$  everywhere in the fluid is same. Also it is assumed that the fluid and the plate is at rest after that the plate is to be moving with a constant velocity  $U_0$  in its own plane and instantaneously at time  $t > 0$ , the temperature of the plate and the species concentration are raised to  $T_w (> T_\infty)$  and  $C_w (> C_\infty)$  respectively, which are thereafter maintained constant, where  $T_w, C_w$  are the temperature and species concentration at the wall and  $T_\infty, C_\infty$  are the temperature and concentration of the species far away from the plate respectively.

The physical model of this study is furnished in the following figure.

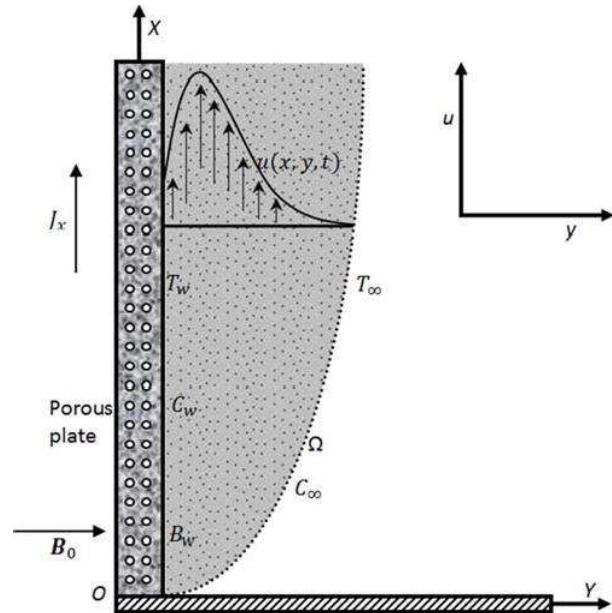


Figure 1. Physical configuration and coordinate system.

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equation

solved numerically by finite difference method. The results of this study will be discussed for the different values of the well known parameters and will be shown graphically.

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \rho \frac{\partial^2 u}{\partial x^2} + \sigma^f B_0^2 u \tag{3}$$

$$\frac{\partial u}{\partial x} = \frac{\partial T}{\partial x} = \frac{\partial (T_\infty + (T_w - T_\infty) f(\eta))}{\partial x} = U_0 (T_w - T_\infty) \frac{\partial f}{\partial \eta}$$

MHD energy equation

$$\frac{\partial T}{\partial y} = \frac{\partial T}{\partial y} = \frac{\partial (T_\infty + (T_w - T_\infty) g(\eta))}{\partial y} = U_0 (T_w - T_\infty) \frac{\partial g}{\partial \eta}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{\partial T}{\partial z} \right) + \frac{\partial}{\partial t} \left( \frac{\partial T}{\partial t} \right) \quad (4)$$

Concentration equation

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) \quad (5)$$

With the corresponding initial and boundary conditions are

$$u = 0, w = 0, T = T_w, C = C_w \text{ at } y = 0 \quad (6)$$

$$u = 0, w = 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \quad (7)$$

## 2. Mathematical Formulation

Since the solutions of the governing equations (1)-(4)

under the initial (6) and boundary (7) conditions will be

based on the finite difference method it is required to make

the said equations dimensionless. For this purpose, now

introduce the following dimensionless quantities;

$$X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{u}{U_0}, V = \frac{v}{U_0}, W = \frac{w}{U_0}, \tau = \frac{t}{L^2}$$

$$\theta = \frac{T - T_w}{T_\infty - T_w}, \bar{C} = \frac{C - C_w}{C_\infty - C_w}$$

$$\rho, U_0, T_w - T_\infty, C_w - C_\infty$$

From the above dimensionless variable

$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial \theta}{\partial \tau}$$

$$\frac{\partial \bar{C}}{\partial \tau} + U \frac{\partial \bar{C}}{\partial X} + V \frac{\partial \bar{C}}{\partial Y} = D \left( \frac{\partial^2 \bar{C}}{\partial X^2} + \frac{\partial^2 \bar{C}}{\partial Y^2} \right)$$

$$\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}$$

$$= \frac{U^2 (T_w - T_\infty) \tau}{P^2}$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + \frac{\partial C}{\partial t}$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + \frac{\partial C}{\partial t}$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + \frac{\partial C}{\partial t}$$

A and

$$\frac{\partial^2 C}{\partial y^2} = \frac{\partial}{\partial y} \left( \frac{\partial C}{\partial y} \right) = \frac{\partial}{\partial y} \left( \frac{U_0 (C_w - C_\infty) \tau}{P} \right) = \frac{U_0 (C_w - C_\infty) \tau}{P}$$

$$\frac{\partial}{\partial y} \left( \frac{U_0 (C_w - C_\infty) \tau}{P} \right) = \frac{U_0 (C_w - C_\infty) \tau}{P}$$

$$= \frac{U^2 (C_w - C_\infty) \tau}{P^2}$$

Now substitute the values of the above derivatives into the

equations (1)-(5) and after simplification obtain the following

nonlinear coupled partial differential equations interms of

$$W, \bar{B} = U \frac{\partial \bar{B}}{\partial X}, \bar{C} = \frac{C - C_w}{C_\infty - C_w}, u = \dots, v = \dots, w = U$$



$$\tau = \frac{\mu H^2 (U_w - U_\infty)}{u \rho (U_w - U_\infty)} \text{ (Dufour Number) .}$$

Also the associated initial (6) and boundary (7) conditions become

$$\tau = 0, W = 0, \bar{\theta} = 1, \bar{\phi} = 1 \text{ at } \tau = 0 \quad (13)$$

$$\tau = 0, W = 0, \bar{\theta} = 0, \bar{\phi} = 0 \text{ as } \tau \rightarrow \infty \quad (14)$$

### 3. Numerical Solutions

To solve the second order non-linear coupled dimensionless partial differential equations (8)-(12) with the

associated initial and boundary conditions (6) and (7) are resolved numerically by using explicit finite difference method. To obtain the difference equations the region of the flow is divided into a grid or mesh of lines parallel to X and Y axes where X-axis is taken along the plate and Y-axis is normal to the plate. Here consider that the plate of height  $X_{max} = 100$  i.e. X varies from 0 to 100 and regard  $\tau_{max} (= 30)$  as corresponding to  $\tau \rightarrow \infty$  i.e. Y varies from 0 to 30. There are  $\tau = 200$  and  $\tau = 200$  grid spacings in the X and Y directions respectively as shown in Figure

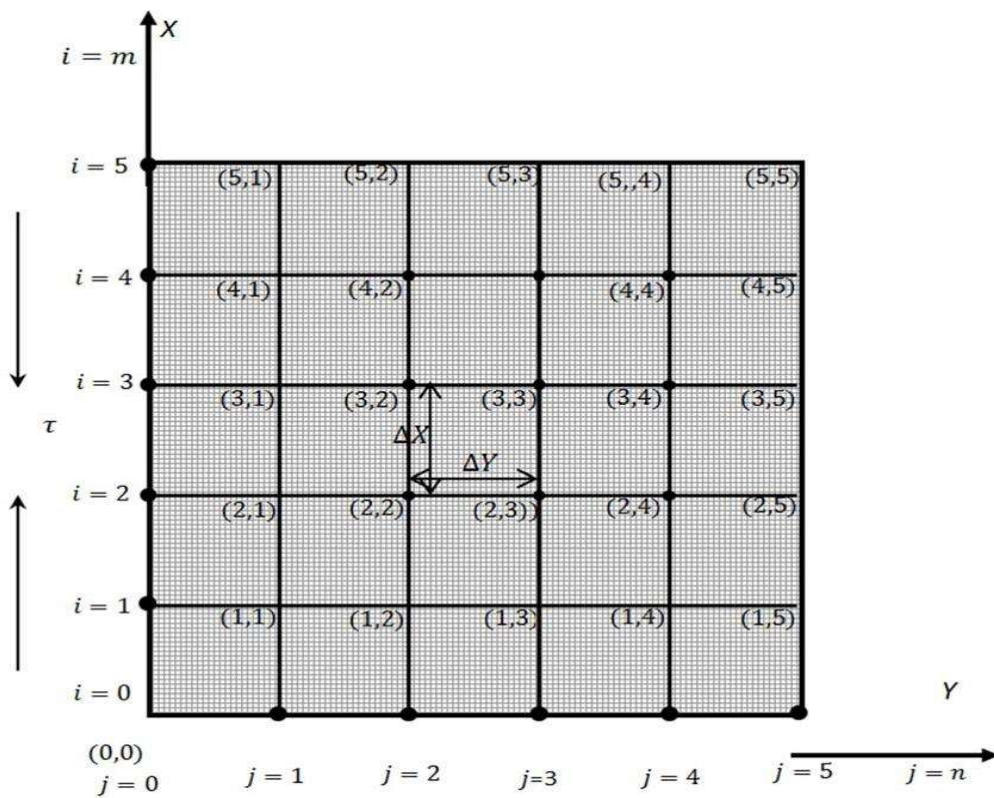


Figure 2. Finite difference space grid.

It is assumed that  $\Delta X, \Delta Y$  are constant mesh sizes along X and Y directions respectively and taken as follows,

$$\Delta X = \frac{H}{200}$$

(16)

From figure 2, it is seen that the primary velocity  $U$  decreases with

and the initial and boundary conditions with the finite difference scheme are

$$U^n = 0, W^n = 0, \bar{T} = 1, \bar{C} = 1 \quad (19)$$

$i,0 \quad i,0 \quad i,0 \quad i,0$

$$U^n = 0, W^n = 0, \bar{T} = 0, \bar{C} = 0 \text{ where } L \rightarrow \infty.$$

$i,L \quad i,L \quad i,L \quad i,L$

Here the subscripts  $i$  and  $j$  designate the grid points with  $x$  and  $y$  coordinates respectively and the superscript  $n$  represents a value of time,  $r = n\Delta r$  where  $n = 0, 1, 2, 3, \dots$ . From the initial condition (19), the values of  $U, W, \bar{T}$  and  $\bar{C}$  are known at  $r = 0$ . During any one time-step, the coefficients  $U_{i,j}$  and  $V_{i,j}$

appearing in equations (15)-(18) are treated as constants. Then at the end of any time-step  $\Delta r$ , the new temperature  $\bar{T}$ , the new concentration  $\bar{C}$ , the new velocity  $U'$  and  $W'$  at all interior nodal points may be obtained by successive applications of equations (15), (16), (17), (18), are respectively. This process is repeated in time and provided the time-step is sufficiently small,

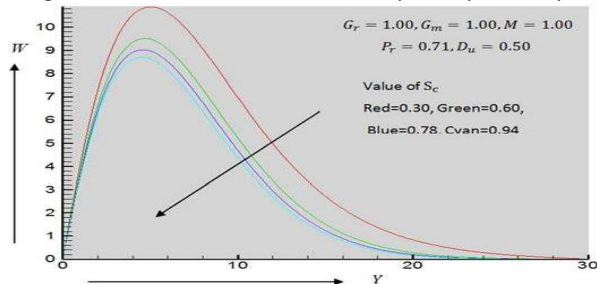
$U, W, \bar{T}$  and  $\bar{C}$  should eventually converge to values which approximate the steady-state solution of equations (8)-(12).

These converged solutions are shown graphically in Figure-3. to Figure-18.

#### 4. Results and Discussion

In order to discuss the results of this problem. The approximate solution are obtain to calculate numerical values of the velocity  $U$ , temperature  $\bar{T}$  and concentration  $\bar{C}$  within the boundary layer for different values of Dufour number  $D_u$ , magnetic parameter  $M$ , Grashof number  $G_r$ , Prandtl number  $P_r$ , Schmidt number  $S_c$  with the fixed value of modified Grashof number  $G_m$ .

To get the steady state solutions, the computations have been carried out up to  $r = 8$ . To observe that the results of the computations, however, changes rapidly after  $r = 45$ . The significance of cooling problem in nuclear engineering in connection with the cooling of reactors,. To investigate the physical situation of the problem, the solutions have been illustrated of Grashof number. The effect of modified Grashof number  $G_m$  on the primary velocity  $U$  is represented in Figure-14. It is observed that the primary velocity  $U$



increases Schmidt number  $S_c$ . The effect of Prandtl number  $P_r$  is represented by Figure-7. we see that the primary velocity  $U$  decreases rapidly with increasing Prandtl

number  $P_r$ . From Figure-10 we observed that the primary velocity  $U$  decreases with increase of magnetic parameter  $M$ .

In Figure-12 the primary velocity  $U$  increases with increase

increases with increase of modified Grashof number  $G_m$ . In Figure-16 represent the effect of the Dufour number  $D_u$  on Primary velocity  $U$ . We observe that the primary velocity  $U$  increases when Dufour number  $D_u$  increases. The secondary velocity profiles have been displayed in Figure -4, 8, 11, 13, 15 and 17. From Figure-4 we observe that the secondary velocity  $W$  decreases with increase of Schmidt number  $S_c$ . In Figure-8. we observe that the Secondary velocity  $W$  decreases with increase of Prandtl number  $P_r$ . The effect of the Magnetic parameter  $M$  on secondary velocity  $W$  is represented by Figure -11. It is observed that the secondary velocity  $W$  increases with increase of magnetic parameter  $M$ . From Figure-13 represent that the secondary velocity  $W$  increases when increases of Grashof number  $G_r$ . In Figure-15 we see that the secondary velocity  $W$  increases with increase of modified Grashof number. From Figure- 17 we see that the secondary velocity  $W$  increases with increase of Dufour number. The temperature profiles have been exhibited in Figure - 5, 9 and 18. From Figure-5. we observe that Temperature  $T$  increases when increases of Schmidt number  $S_c$ .

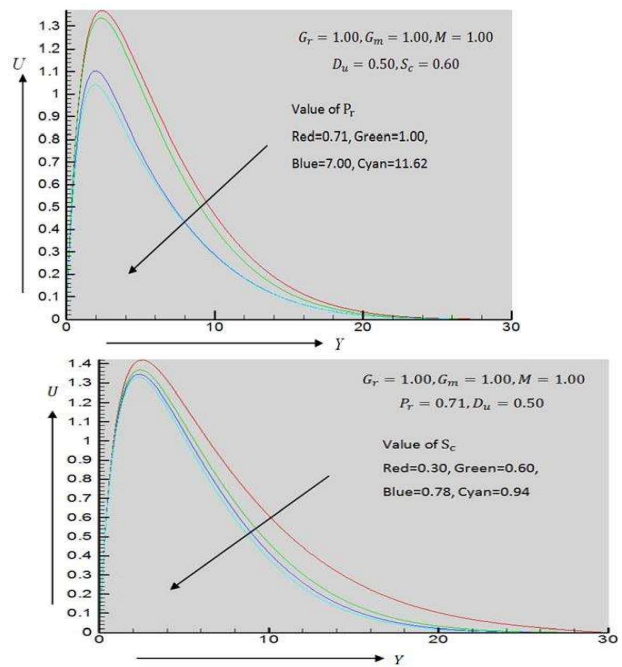


Figure 3. Primary velocity profile due to change of Schmidt number.

Figure 4. Secondary velocity profile due to change of Schmidt number.

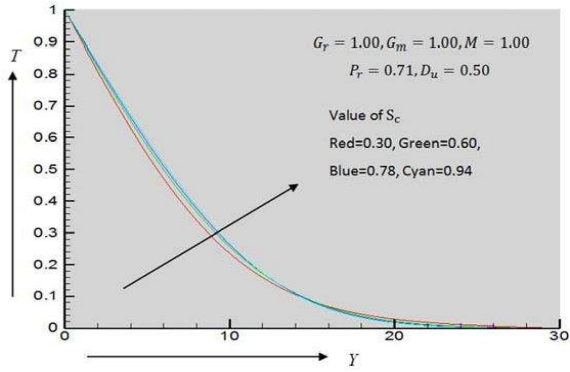


Figure 5. Temperature profile due to change of Schmidt number

Figure 6. Concentration profile due to change of Schmidt number.

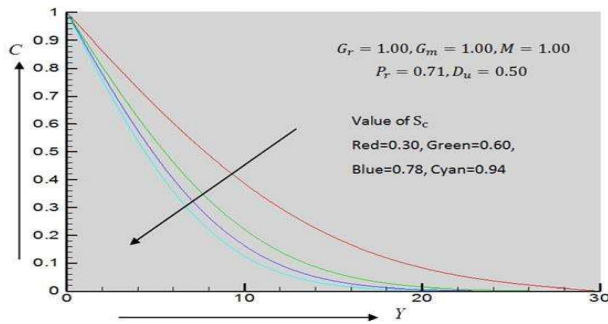


Figure 7. Primary velocity profile due to change of Prandtl number.

### 5. Conclusions

In the present research work, the heat and mass transfer effects on MHD free convection fluid flow past a vertical porous plate. The results are given graphically to illustrate the variation of velocity, temperature and concentration with

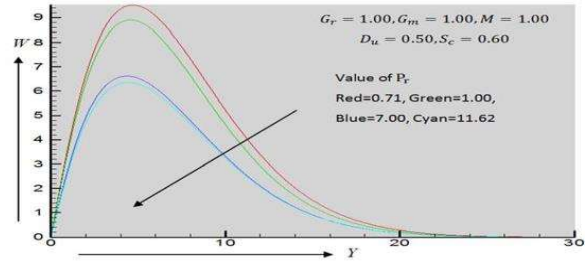


Figure 8. Secondary velocity profile due to change of Prandtl number.

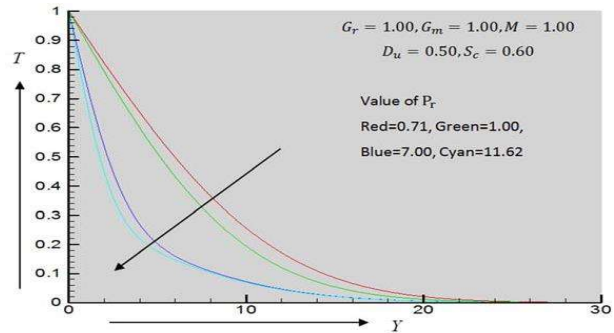


Figure 9. Temperature profile due to change of Prandtl number

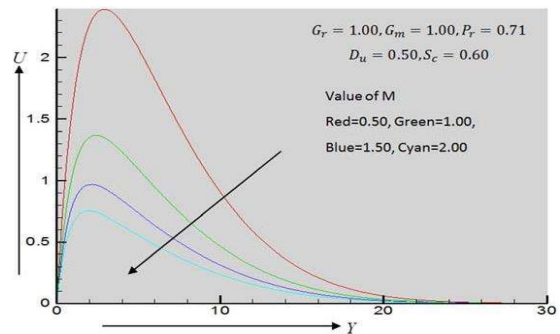


Figure 10. Primary velocity profile due to change of Magnetic parameter.

different parameters, Important findings of this investigation are given below:

The primary velocity profiles  $U$  decreases with the increases of Schmidt number ( $S_c$ ) Prandtl number ( $P_r$ ) and Magnetic parameter ( $M$ ). On the other hand primary velocity profiles  $U$  increases with the increases in Grashof number ( $G_r$ ), modified Grashof number ( $G_m$ ) and Dufour number ( $D_u$ ). The Secondary velocity profiles  $W$  decreases with the increases of



Schmidt number ( $S_c$ ) and Prandtl number ( $P_r$ ) as well as reverse effect with the increases of Grashof number ( $G_r$ ), modified Grashof number ( $G_m$ ) and Dufour number ( $D_u$ ) and Magnetic parameter ( $M$ ). The temperature increases with the increases of Schmidt number ( $S_c$ ) and Dufour number ( $D_u$ ). Whereas it decreases with an increase of Prandtl number ( $P_r$ ). The Concentration  $C$  decreases with the increases of Schmidt number ( $S_c$ )

### Conflict of Interest

The authors declare that they have no any conflict of interest.

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